BVG Trust

J. Sci. Trans. Environ. Technov. 2014, 8(1) : 50-54

Scientific Transactions in Environment and Technovation

Behaviour of oil spilled under solid ice cover

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Abstract

The flow of oil spreading under solid ice covered sea is considered. The flow dynamics are governed by the continuity and Navier-Stokes equations coupled to a species equation for the concentration of oil. The effective dispersion coefficient was analyzed and found that there was a decrease in dispersibility.

Keywords: Concentration of oil, Oil spill, Sea ice, Taylor dispersion model.

INTRODUCTION

Spreading of oil in the presence of an ice cover makes the sea ice environment more unpredictable. Even though several authors analyzed spreading of oil in ice-infested waters, Glaeser and Vance (1971) were pioneers to measure the oil spreading under solid ice. Hoult et al. (1975) studied extensively the oil spreading on and under ice, including an analysis of surface characteristics of arctic ice. Chen et al. (1976) derived two equations for oil spreading under ice covers by using the equilibrium condition between buoyancy and viscous forces.

Yapa and Chowdhury (1990) presented a set of equations to describe oil spreading under ice from the time of beginning of a spill to termination. The equations were derived using a simplified form of the Navier-Stokes equations and verified through experiments using oils of different viscosities, ice covers of different roughness and a variety of discharge conditions. They proposed that as and when oil is under ice, buoyancy is the spreading force, which is equivalent to gravity in surface water spreading. Alhimenko et al. (1997) developed a three dimensional numerical model to simulate oil pollution under solid and broken ice. Their study included the effects of coriolis force, eddy viscosity, atmospheric pressure, advection and diffusion, buoyancy and non-conservativity.

Yapa and Dasanayaka (2006) reviewed the state-of-the-art in modeling the spread and movement of oil in ice. The finite difference approach for modeling the motion of viscous oil slicks among ice floes has beenrecommended (Gjosteen (2004)). Buist et al. (2008) reported that the oil spilled within the pack ice generally move along with ice. In open drift ice, the oil and ice may move at different rates and directions under the variable influence of winds and currents. Bellino et al. (2013) investigated the prediction model for oil spread inside an ice channel and found that *in-situ* burning is effective for removing the oil, when the slick layer spilled is thicker.

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Oil spread under the ice covered sea is a complex process, whose description requires accounting for a large number of various factors. In this study, the movement of oil spilled under solid ice covered sea hasd been considered . This model predicts the transport of oil spill using Eulerian approach during calm weather, without the influence of waves and water current. All the thermo physical properties are assumed to be constant in the linear momentum equation. The species equation is solved for effective dispersion coefficient and concentration using Taylor dispersion model provided by Taylor (1953) which is the main objective of the study. The dispersion theory developed by Taylor has been widely accepted, which is extended to consider the dispersion phenomena for a wide variety of flows which are too complex to solve analytically (Kumar et al.,2011; Meena Priya and Nirmala P.Ratchagar, 2012; and Rudraiah and Raghunatha Reddy,2013;).

Mathematical Formulation

We consider a steady two- dimensional laminar incompressible viscous flow of oil spreading under solid ice covers on an infinitely long horizontal channel bounded above by impermeable solid ice and below by moving water. No layer of water exists in between oil and solid ice. Oil is considered as a continuum in the model formulation. Effect of coriolis force which is significant in ocean is included as it acts on oil because of its movement. The governing equations based on the conservations of mass and momentum using simplified form of the Navier-Stokes equations are given as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + fv \quad (2)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - fu \quad (3)$$

where, u, v are the components of velocities along the horizontal perpendicular axes x and y, respectively, w is the velocity along the vertical axis z (positive upwards), ν is the kinematic viscosity, P is the pressure, ρ is the density and f is the coriolis parameter(Cushman-Roisin and Beckers(2011)).

Under the assumptions that there exist no pressure gradients and no spatial gradients in velocities, the terms $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}$, w are all identically zero (Ekman(1905)). Thus the governing equations reduces to

$$\nu\frac{\partial^2 u}{\partial z^2} + fv = 0 \quad (4)$$

$$\nu\frac{\partial^2 v}{\partial z^2} - fu = 0 \quad (5)$$

The appropriate boundary conditions for the velocity field are

$$u = u_0 , v = 0 \quad at\ z = 0$$

$$u = 0 , v = 0 \quad at\ z = h \quad (6)$$

where, h is the oil slick thickness.

Introducing the following non-dimensional quantities

$$u^* = \frac{u}{u_0} ;\ v^* = \frac{v}{u_0} ;\ z^* = \frac{z}{h}$$

equations (4) and (5), neglecting the (*) symbol becomes

$$\frac{\partial^2 u}{\partial z^2} + \frac{1}{Ek}v = 0 \quad (7)$$

$$\frac{\partial^2 v}{\partial z^2} - \frac{1}{Ek}u = 0 \quad (8)$$

where, $Ek = \frac{\nu}{f\,h^2}$ is the Ekman number.

The boundary conditions (6) in non-dimensional form are

$$u = 1 , v = 0 \quad at\ z = 0$$

$$u = 0 , v = 0 \quad at\ z = 1 \quad (9)$$

Method of Solution

Velocity

Equation (7) and (8) which are simultaneous linear second order differential equations are solved analytically along with the boundary conditions (9) gives the velocities as

$$u(z) = e^{\lambda z}(g_1 Cos\,\lambda z + g_2 Sin\,\lambda z) + e^{-\lambda z}(g_3 Cos\,\lambda z + g_4 Sin\,\lambda z) \quad (10)$$

$$v(z) = -e^{\lambda z}(g_2 Cos\,\lambda z - g_1 Sin\,\lambda z) + e^{-\lambda z}(g_4 Cos\,\lambda z - g_3 Sin\,\lambda z) \quad (11)$$

where, $\lambda = \frac{1}{\sqrt{2\,Ek}}$ and the coefficients g_i(i = 1,2,3,4) are given in the appendix.

The average velocity given by,

$$\bar{u} = \int_0^1 u(z)dz$$

$$= \frac{g_1 e^{\lambda}}{2\lambda}(Cos\lambda + Sin\lambda) + \frac{g_1 e^{-\lambda}}{2\lambda}(Cos\lambda - Sin\lambda) + \frac{g_2 e^{\lambda}}{2\lambda}(Sin\lambda - Cos\lambda) - \frac{g_2 e^{-\lambda}}{2\lambda}(Cos\lambda + Sin\lambda) + \frac{e^{-\lambda}}{\lambda}(Sin\lambda - Cos\lambda) + \frac{1}{\lambda}(g_2 - g_1 + 2) \quad (12)$$

gives the relative velocity V as, $V = u - \bar{u}$

Concentration

The concentration C of oil under solid ice is defined by the species equation of the form

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) \quad (13)$$

where, D is the molecular diffusivity assumed to be a constant.

Assuming that the flow is uniform, steady and unidirectional and the horizontal variations are smaller than the vertical variation (i.e. $\frac{\partial^2 C}{\partial x^2}, \frac{\partial^2 C}{\partial y^2} \ll \frac{\partial^2 C}{\partial z^2}$), the concentration C of oil under solid ice in the reduced form is

$$u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial z^2} \quad (14)$$

We now introduce the following dimensionless groups to equation (14)

$$x^* = \frac{x}{L} ;\ z^* = \frac{z}{h} ;\ c^* = \frac{c}{c_0} \quad ; ;$$

and obtain $\frac{V}{L}\frac{\partial c}{\partial \zeta} = \frac{D}{h^2}\frac{\partial^2 c}{\partial z^2}$ (15)

where, $\zeta = \frac{x - \bar{u}t}{L}$ is the dimensionless axial moving coordinate.

The boundary conditions for concentration given by,

$$c = c_0 \ at\ z = 0$$

$$\frac{\partial c}{\partial z} = 0 \ at\ z = h$$

becomes, $c = 1 \ at\ z = 0$

$$\frac{\partial c}{\partial z} = 0 \ at\ z = 1 \quad (16)$$

in non-dimensional analysis.

The solution to equation (15) satisfying the conditions (16) gives the concentration distribution as the variation of z as,

$$C = Q\left\{\frac{e^{\lambda z}}{2\lambda^2}(g_1 Sin\lambda z - g_2 Cos\lambda z) + \frac{e^{-\lambda z}}{2\lambda^2}(g_1 Sin\lambda z + g_2 Cos\lambda z) - \frac{e^{-\lambda z}}{2\lambda^2}(Sin\,\lambda z) - \frac{\bar{u}\,z^2}{2} + g_5 z + g_6\right\}$$

where, $Q = \frac{h^2}{DL}\frac{\partial c}{\partial \zeta}$ and the coefficients g_i(i = 5,6) are given in the appendix.

Dispersion Coefficient

The volumetric rate at which the fluid is transported across a section of a layer of unit breadth is

$$M = \int_0^h C\,V\,dz = h\int_0^1 C\,V\,dz$$

Solving we get,

$$M = \frac{h^3}{DL}\,G\,\frac{\partial c}{\partial \zeta} \qquad (17)$$

where,

$$G = I_1 + I_2 + I_3 + I_4 + I_5 + I_6 + I_7 + I_8 + I_9 + I_{10} + I_{11} + I_{12}$$

The coefficients $I_i(i = 1,2,3,\ldots 12)$ are given in the appendix.

Based on the observation by Taylor(1953), we assume that the variation of C with are small compared with those in the longitudinal direction and ζ is the dimensionless cross-sectional average concentration, $\frac{\partial c}{\partial \zeta}$ is indistinguishable from $\frac{\partial C_m}{\partial t}$ so that (17)

can be written as $M = \frac{h^3}{D\,L}\,G\,\frac{\partial C_m}{\partial \zeta}$ (18)

The fact that no material is lost in the process is expressed by the continuity equation for namely

$$\frac{\partial M}{\partial \zeta} = \frac{-2}{L}\,\frac{\partial C_m}{\partial t} \qquad (19)$$

where, $\partial/\partial t$ represents the differentiation w. r. t. time at point where ζ is constant.

Differentiating equation (18) w. r. t . and equating to (19) gives

$$\frac{\partial C_m}{\partial t} = D^*\,\frac{\partial^2 C_m}{\partial \zeta^2}$$

where, $D^* = \frac{-h^3}{2\,D}\,G$

gives the effective dispersion coefficient governing the longitudinal dispersion. Equation (20) implies that C_m is dispersed relative to a plane which moves with average velocity exactly as though, it has been diffused by a process which obeys the same law as the molecular diffusion.

RESULTS AND DISCUSSION

Numerical evaluation of the analytical results for the velocity, effective dispersion coefficient and concentration has been computed. Due to the coriolis effect, the characteristics of geophysical flows vary greatly with the values of Ekman number. Thus the results presented are discussed for different values of Ekman number.

Figure 1 shows the velocity profile for different Ekman number. It indicates that a rise in Ekman number promotes the velocity of oil.

Dispersion of oil is studied using Taylor dispersion model. The effective dispersion coefficient computed for different Ekman number is discussed through Figure 2 to study the oil characteristics. From the figure we see that the Ekman number retards the dispersion coefficient D*.

This decrease in dispersibility is because of the fact that the thicker slick becomes more viscous and the energy required is increased to tear it into small droplets. Dispersion coefficient of the under ice slick is much smaller than in open waters because the ice sheet weakens the turbulent wave and the friction also restricts the slick from elongating to some extent.

We extend our study for oil concentration, which provide necessary information to understand the transport of oil. Figure 3 shows concentration where in Ekman number enhances the oil concentration.

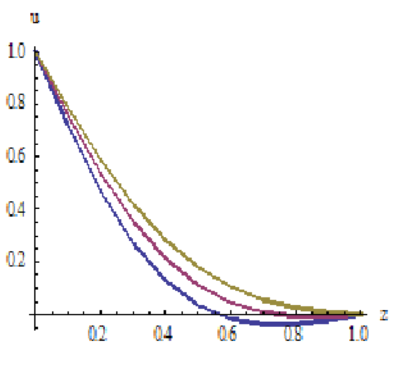

Figure 1: Effects of Ekman number on velocity profile

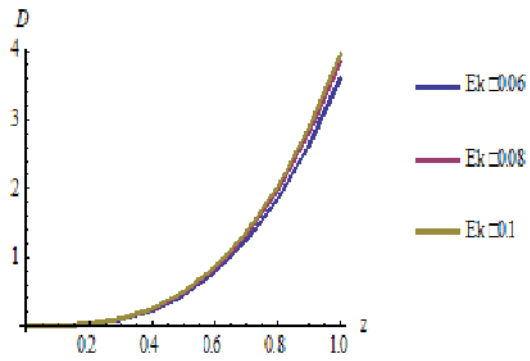

Figure 2: Effective dispersion coefficient varying for different Ekman number

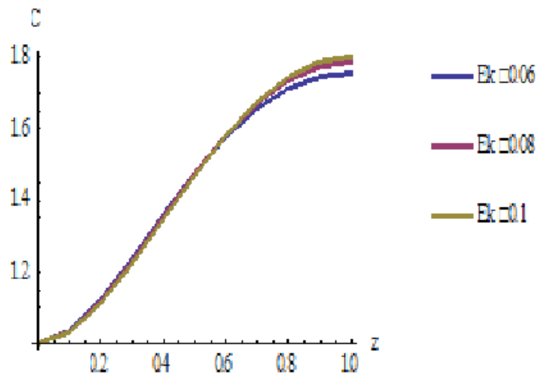

Figure 3: Concentration of oil particle varying for different Ekman number

CONCLUSION

Movement of oil spilled under ice is difficult to understand, since the oil is hidden from view beneath a thick sheet of ice. The present model is developed to predict the movement of oil slick under solid ice cover, effective dispersion coefficient and the distribution of oil particle concentration. The importance for practice of the diffusion analysis of Taylor and the subsequent investigations lies in the ability of the transport equation to take into account complicated velocity and concentration profiles in a simpler manner, as well as providing a theoretical framework for the dispersion coefficient.

ACKNOWLEDGMENT

The University Grants Commission, New Delhi is gratefully acknowledged for the financial support. [BSR grant no.: F.4-1/2006(BSR)/7-254/2009(BSR)].

Appendix

$$g_1 = \frac{-e^{-\lambda}(Sinh\,\lambda\;Cos^2\,\lambda\; - Cosh\,\lambda\;Sin^2\,\lambda\,)}{2\,(Sin^2\,\lambda\; + \;Sinh^2\,\lambda\,)}$$

$$g_2 = \frac{-\;Cos\,\lambda\;Sin\,\lambda\,)}{2\,(Sin^2\,\lambda\; + \;Sinh^2\,\lambda\,)}$$

$$g_3 = 1 - g_1$$

$$g_4 = g_2$$

$$g_5 = \frac{e^{-\lambda}}{2\,\lambda}(Cos\lambda - Sin\lambda) + \left(\frac{g_2 - g_1}{\lambda}\right)(Cos\,\lambda\,Cosh\,\lambda) - \left(\frac{g_1 + g_2}{\lambda}\right)(Sin\,\lambda\,Sinh\,\lambda) + \bar{u}$$

$$g_6 = \frac{1}{Q}$$

$$I_1 = \frac{({g_1}^2 - {g_2}^2)}{16\,\lambda^3}\left[e^{2\lambda}(-Cos2\lambda + Sin2\lambda) + 1\right]$$

$$I_2 = \frac{({g_2}^2 - {g_1}^2 + 2g_1 - 1)}{16\,\lambda^3}\left[-e^{-2\lambda}(Cos2\lambda + Sin2\lambda) + 1\right]$$

$$I_3 = \frac{g_1 g_2}{\lambda^2}$$

$$I_4 = \frac{-g_1 g_2}{8\lambda^3}\left[e^{2\lambda}(Cos2\lambda + Sin2\lambda) - 1\right]$$

$$I_5 = \frac{-g_1 g_2}{8\lambda^3}\left[e^{-2\lambda}(-Cos2\lambda + Sin2\lambda) + 1\right]$$

$$I_6 = \frac{g_2}{8\lambda^3}\left[e^{-2\lambda}(-Cos2\lambda + Sin2\lambda) + 1\right]$$

$$I_7 = -\frac{g_2}{\lambda^2}$$

$$I_8 = \left(\frac{-g_2\,\bar{u}}{2} + g_2 g_5 + g_2 g_6 - \frac{g_1\bar{u}}{2\lambda^2}\right)\left[\frac{e^{\lambda}}{2\lambda}(-Cos\lambda + Sin\lambda)\right] + \left(\frac{g_2\,\bar{u}}{4\lambda^3}\right)\left[e^{\lambda}(Cos\lambda + Sin\lambda) - 1\right] - (g_2\bar{u} - g_2 g_5)\left[\frac{e^{\lambda}}{2\lambda^2}Cos\lambda\right] + \left(\frac{g_2 g_6}{2\lambda} - \frac{g_1\bar{u}}{4\lambda^3} - \frac{g_2 g_5}{2\lambda^2}\right)$$

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